Meyer and Mendelejeff, and dissertations by Liebig, Bunsen, Pasteur, Berzelius, Davy, etc.

In the case of many of the older investigators, the editors of the Series have reproduced only what they deemed important. The Series is not, therefore, in all cases a *full* reprint of the scientific classics. It might have been desirable, further, to print the texts of the originals along with the German translations. Although probably not warranted from a commercial point of view, this step would have made the Series international in its character and usefulness. Altogether, we cannot close without words of high commendation for the undertaking, nor without expressing the hope that its range of usefulness will be extensive and its fruits beneficent.

THOMAS J. MCCORMACK.

VORLESUNGEN UEBER GESCHICHTE DER MATHEMATIK. By *Moritz Cantor*. Leipsic: B. G. Teubner. 1894–1896. Price, Vol. I., 22 Marks; Vol. II., 24 Marks; Vol. III., Two Installments, 12 Marks.

It would be impossible to do justice to this monumental work within the brief limits of a book review, even if the task were not rendered supererogatory by the high standing of the work and the acknowledged authority of its author. Cantor's *Lectures on the History of Mathematics* are the work of a man who has unswervingly devoted a life-time to this single task, who thirty-three years ago was well known for his important contributions to this subject, and who can now in the second edition of the first volume of his great work point with pride to the impulse and awakened interest which his endeavors have aroused in the historical studies of his science. He has had many predecessors, each of whom has distinguished himself in certain branches and by certain excellences—Montucla who excelled in lucidity, elegance, and popularity; Libri who seems to have united in an eminent degree all the qualities necessary to the makeup of a writer of a universal mathematical history, but whose work extends only to the period preceding Galileo in Italy; Hankel, whose contributions to the history of early mathematics are marked by much acumen; and several others. Nevertheless, it may safely be said that profundity, accuracy, and extensiveness of treatment have never before in any history of mathematics been so thoroughly and intimately united as in the three volumes constituting these *Lectures* of Moritz Cantor. The first volume embraces the period from earliest antiquity to the year 1200 A. D. and is now in its second edition, thoroughly revised and brought down to date (1894). The second volume embraces the time from 1200 to 1668 A. D. The third and last volume will comprise the time from 1668 to 1759, concluding with the first epoch-making papers of Lagrange in the Proceedings of the Turin Academy. The first two installments only of this third volume have appeared (1894–1896), the third is still in preparation.

In the Introduction to Volume I., which contains 883 pages with a chart of ancient numerical characters, we have some brief philosophical considerations concerning the psychological origin of mathematical operations and the invention

of numerical signs. As to the theory that the first numerical words originally denoted not numbers but definite objects, Prof. Cantor remarks that philology has not succeeded in proving its position. Nor can he himself offer much to the solution of the problem. We are on sure ground, he says, only when we come to derivative numerical words. We have also some interesting remarks on the various systems of numbers, namely, the decimal, vigesimal, undecimal, sexigesimal systems, etc. The true history of mathematics, the author contends, begins only with the first written monuments and inscriptions which are presumably found in Egypt. 55 pages are devoted to the mathematics of Egypt, 31 to that of the Babylonians, 65 to that of the Indians, 29 to that of the Chinese, and 118 to that of the Arabs. The remaining three divisions of the first book are devoted to the mathematical achievements of the Greeks, which naturally take up the largest space, and to those of the Romans and of the early mediæval monasteries. The researches of the ancient nations are extremely interesting, not only from the point of view of mathematical history but equally so from that of philosophy and psychology. Their insight and errors are of extreme importance, and it is both profitable and fascinating to witness the primitive operations of the human mind as employed upon this its surest and most fundamental subject. Of the Greeks the most interesting chapters are those relating to Pythagoras and Archimedes. Dr. Cantor gave long ago, in his *Mathematische Beiträge zum Kulturleben der Völker*, 1863, a charming appreciation of the life and achievements of Pythagoras, only differing from the chapter on the great philosopher in the present work by being more popular and less exhaustive. In Archimedes we have the man who may be regarded as the incarnation of the mathematical genius of antiquity, and the chapter devoted to him shows at its best the precious heritage which he left to us. It is surprising to note to what a pitch the Indians advanced arithmetic and algebra, and also to follow the work of the Arabs. In fine, the entire first volume is a book which can be read and consulted by writers of average elementary mathematical attainments, and offers material from which all readers may draw profit and entertainment.

The year 1200 was an important one in the history of European Mathematics, and is fitly chosen as the beginning of the second volume. Christianity was then in possession of the art of arithmetic, as it had been recovered from its different ancient and Eastern sources. It was also in possession of the zero and of the the no less important principle of the positional value of figures. Algebra, as far as equations of the first and second degree, had been compassed, the geometry of Euclid, the astronomy of Ptolemy, the writings of Theodosius, and of Menelaus, existed in Latin translations, and appositely to the right time came the right men who were destined to achieve great things in mathematical science,—Leonardo of Pisa and Jordanus Nemorarius.

"Leonardo," says Cantor, "was a practised arithmetician and geometer, an ingenious algebraist, conversant with the application of algebra to geometry, as well as a creative genius of high rank in the theory of numbers." Jordanus

Nemorarius was a priest and member of a powerful order; he fell little short of Leonardo in point of mathematical ability, but by reason of his ecclesiastical position his influence was more powerful and decisive than that of the other who was a merchant. From these two great landmarks the second volume traces the history of mathematics through the early developments of algebra and geometry in England, France, Italy, and Germany, including Nicolaus of Cusa, Regiomontanus, Leonardo da Vinci, Luca Paciuolo, Michael Stifel, etc., down to the researches on cubic equations by Cardano and Tartaglia, where the first installment ends. The second installment is devoted to the advances made in cyclometry and trigonometry by Vieta, Van Roomen, etc., to the researches on equations of the fourth degree by Bombelli, etc., Kepler's and Pascal's investigations in geometry, the rise of mechanics, logarithms, continued fractions, the theory of numbers, analytical geometry, and lastly to the germs of the infinitesimal calculus in Kepler, Cavalieri, and most notably of all in Fermat. The volume concludes with the year 1668–1669, a momentous epoch in the history of mathematics, for at that time Gottfried Wilhelm Leibnitz was publishing at Leipsic his Doctor's dissertation, and Isaac Newton had just been elected to the chair of Mathematics in Cambridge University England.

With this epoch the second volume begins. The period which follows is of all that of most import for modern mathematics, and its utterances are associated with the most interest for professional readers. The first installment deals with the "geometrical character" of Leibnitz, with certain developments of commercial arithmetic, with the history of series as developed by Mercator, Brouncker, Gregory, Newton, Leibnitz, Halley, De Moivre, James Bernoulli, with continued fractions, the theory of curves, etc. We have also in this installment a chapter on Newton and Leibnitz's first discoveries in the domain of the infinitesimal calculus, chapters on Leibnitz and on the brothers Bernoulli both preceding and during their famous strife. The great controversy concerning the priority of invention of the differential calculus between the followers of Newton and Leibnitz,—a controversy which excited the mathematical world for more than twenty-five years, and which was really not definitively settled until the present century,—takes up a good part of the second and latest installment of Cantor's third volume. There is now, of course, little to be said upon the subject of this controversy, and Cantor does not claim to add much to its elucidation, except to point out an omission made in the copying of a letter by Leibnitz to Wallis of the word *hodie*, which might easily have led to certain suspicions in the English mind as to Leibnitz's fair dealings. His conclusion is that now that both great inquirers have received their just share of the credit owing to them for their discoveries, a careful and unprejudiced examination of the controversy unfortunately shows that the conduct of the matter reflected no little discredit upon *all* parties concerned. The last installment closes with the developments of the calculus, of Algebra, and of analytical and projective geometry down to the year 1726. The work of Euler and his period remains.

Cantor's history now comprises 2218 pages. The final installment, reaching to the year 1759, and which is yet to appear, will certainly not increase its bulk to much over 2600 pages, leaving the vast material from the date of Lagrange's first memoirs on to be elaborated by another hand. The history will thus hardly exceed in size some of its predecessors, but it will contain proportionately more material, from its being almost exclusively devoted to the solider scientific aspects of its subject and not so much to biographical and personal details, which served so greatly to swell the work of Montucla. In fine, it is far and away the concisest, yet most comprehensive and authoritative treatment of the subject that we have. As such, it is the indispensable adjunct of every mathematical worker and absolutely necessary in every mathematical library. T. J. McC.

PHYSIKALISCH-CHEMISCHE PROPAEDEUTIK. Unter besonderer Berücksichtigung der medicinischen Wissenschaften und mit historischen und biographischen Angaben. Von *Professor Dr. Med. et Phil. H. Griesbach.* Erste Hälfte. 272 Pages. Price, M. 6. Zweite Hälfte, I. Lieferung, 320 Pages. Price, M. 7. Leipsic: Wilhelm Engelmann. 1895 and 1896.

The present work is in the nature of an encyclopædic introduction to medicine, and deals with the specific chemical and physical facts, as well as methods, which enter into the foundations and structure of that science. The work is published in two parts, comprising three installments of some 300 pages each, and covers an unusually vast field. Its author is a man of scientific attainments and of wide and profound bibliographical knowledge. He has materially added to the attractiveness of the work by interweaving with his expositions a great mass of biographical and historical data. Each subject treated acquires thus a developmental form, well adapted to strengthening the memory of the student for the different subjects. Altogether, we have in the book an abridged history of science, and even of philosophy, the main subjects of which are also incidentally touched upon. Since the work presumes no special scientific or mathematical knowledge, it may be used with profit by every student, no matter what his profession or sphere of activity, the material it offers being such as should be known by every educated member of society. Further, on all the subjects coming within the designation of the "propædeutics of physics and chemistry" it constitutes a valuable reference book of the facts, and more especially of the literature, as also an etymological dictionary of scientific terms. That many dubious philosophical considerations should have slipped into a work which covers so vast a field and sounds the depths of so many sciences is natural and intelligible. This we shall see in the following review of the contents:

We have in Chapter I. a discussion of the character of science and logic; in Chapter II. a discussion of the character, method, and aim of physical science; Chapter III. treats of the origin of physical and chemical science and of scientific observation; in Chapter IV. space and time are treated. Here the author takes the